ECE 343: Signals and Systems

Homework 5: Fourier Transforms on $\mathbb{R}$

Please use the regular frequency definition of the Fourier Transform (the one in the handout and below), rather than the book's radian frequency definition.

$$\mathcal{F}\{x(t)\} = X(f) = \int_{-\infty}^{\infty} x(t)e^{-2\pi jft}dt$$

1. Prove the equations relating the book's radian frequency Fourier transform to the regular frequency Fourier transform. Recall that $\omega = 2\pi f$.

$$X_{rad}(\omega) = X\left(\frac{\omega}{2\pi}\right)$$

$$X(f) = X_{rad}(2\pi f)$$

2. Text 7.1-3.

3. Text 7.1-5.

4. Given that $\mathcal{F}\{e^{j\pi t^2}\} = \frac{1+j}{\sqrt{2}}e^{-j\pi f^2}$, find

$$\int_{-\infty}^{\infty} \cos(t^2)dt$$

and

$$\int_{-\infty}^{\infty} \sin(t^2)dt.$$

These are called Fresnel integrals, and are important in the theory of diffraction.

5. Differential equation.

   (a) Use Fourier transforms to write the solution to the differential equation

$$\frac{dy}{dt} + y = x$$

   in terms of a convolution involving $x(t)$.

   (b) Solve the equation (i.e., find $y(t)$) when $x(t) = \Pi(t - \frac{1}{2})$.

6. The convolution of two Gaussians is another Gaussian. Find μ and σ in terms of μ_1, σ_1, μ_2, and σ_2, which are all real and $\sigma_i > 0$.

$$\frac{1}{\sqrt{2\pi\sigma_1^2}}e^{-(t-\mu_1)^2/(2\sigma_1^2)} * \frac{1}{\sqrt{2\pi\sigma_2^2}}e^{-(t-\mu_2)^2/(2\sigma_2^2)} = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-(t-\mu)^2/(2\sigma^2)}$$